Simple graph reduction with visualisation

Outline

- The miniFP language
- Graph representation of terms
- Tying the knot
- The compiler
- The reduction engine
- Literature

The miniFP language

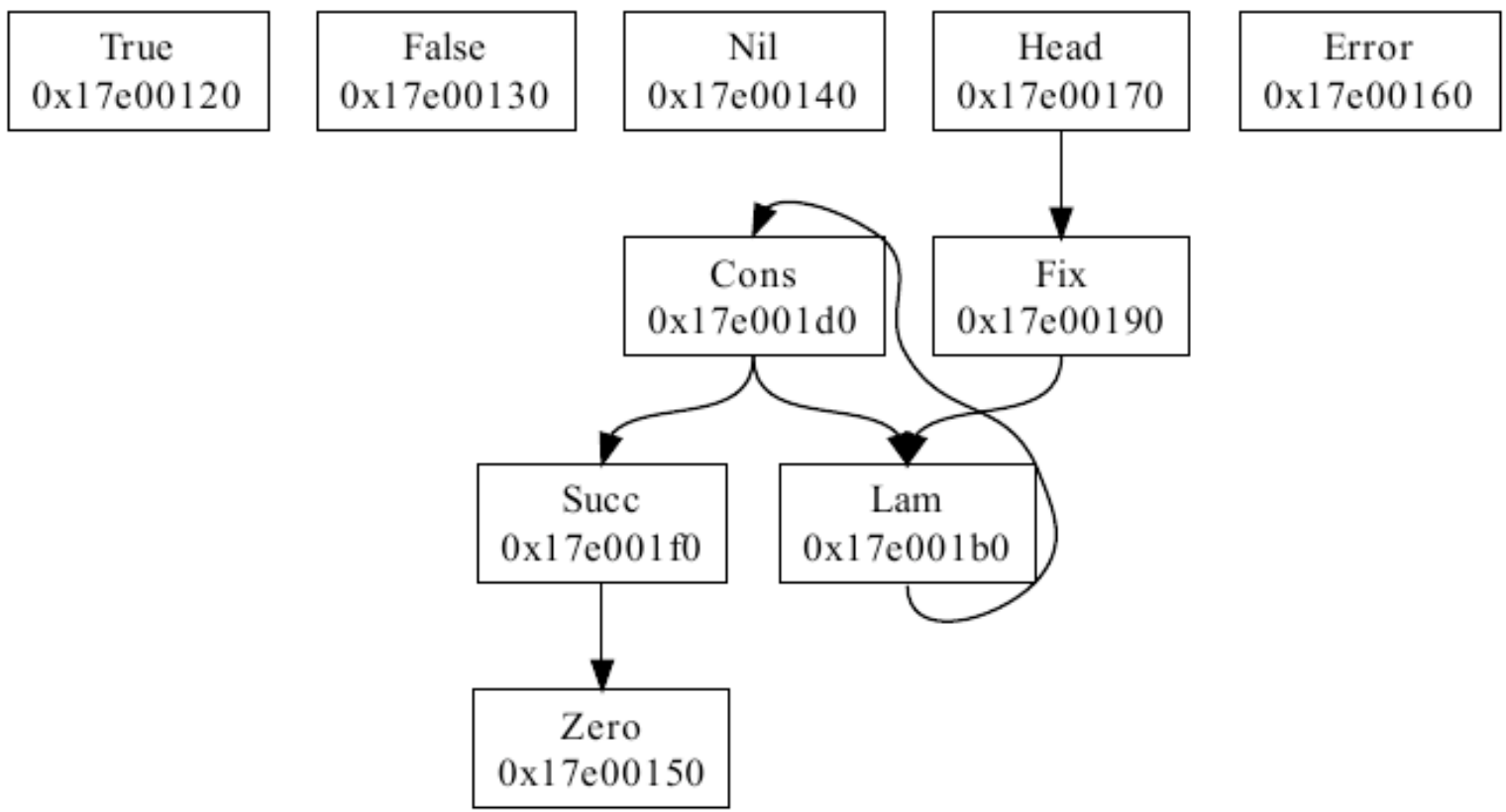
- Small functional language, intended for teaching. Ugly syntax borrowed from Schwartzbach's notes (easy to parse, but annoying to use).
- Hindley/Milner style type inference, with let polymorphism.
- Data types: Bool, Int, List, ->
- No user defined types.
- Explicit fixed-point operator (fix).
- Minimal primitives (head, tail, cons, nil, zero, true, false, pred, succ, isZero, null, if-then-else).

The miniFP language

```
let map = fix (\m -> \f -> \list ->
    if null list
       then nil
       else cons (f (head list)) (m f (tail list))
    fi
    end end end)
in map (\x -> succ x end) [0,1,2] end
```

Graph representation of terms

```
head (fix (\rec -> cons 1 rec end))
```



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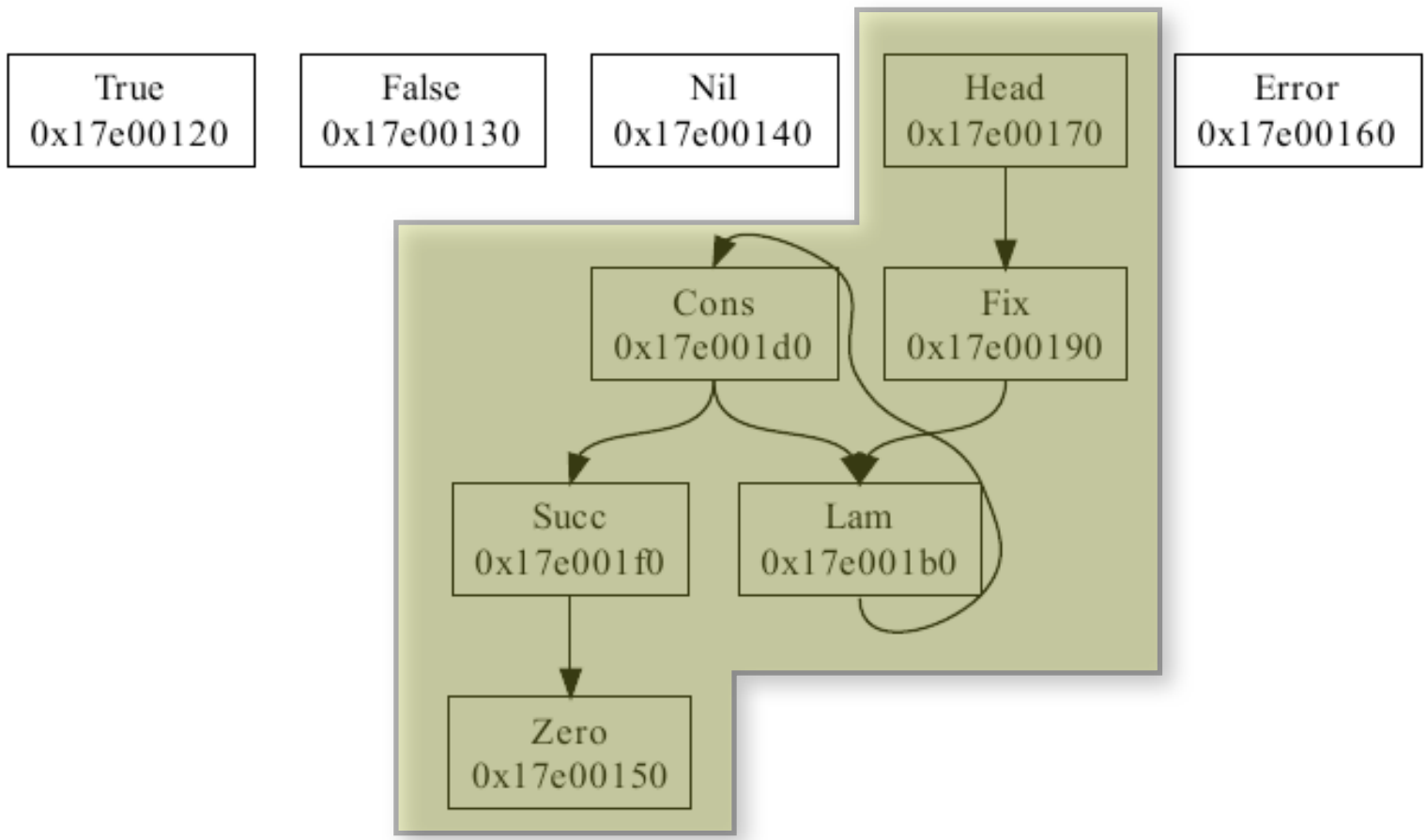

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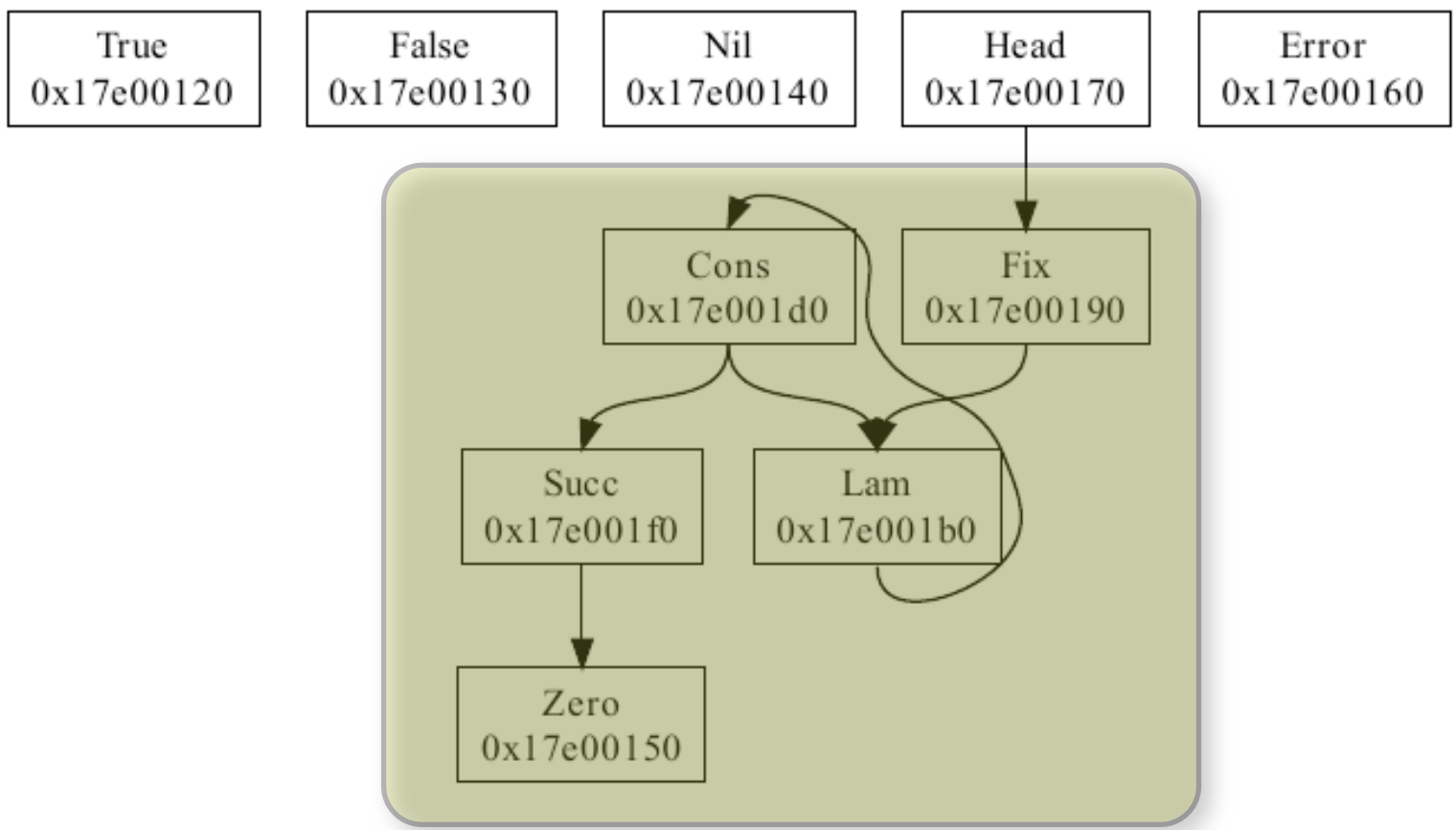

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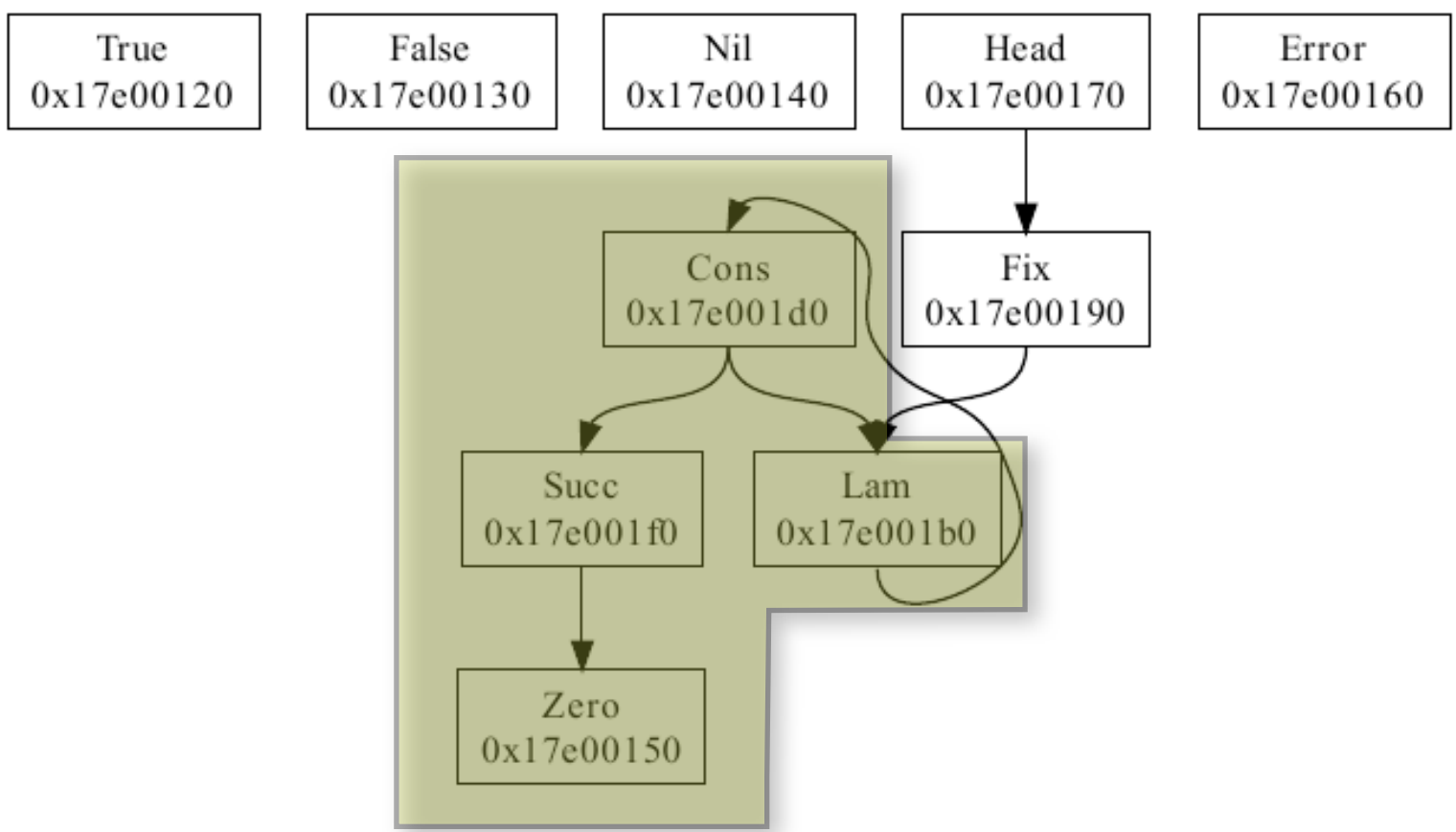

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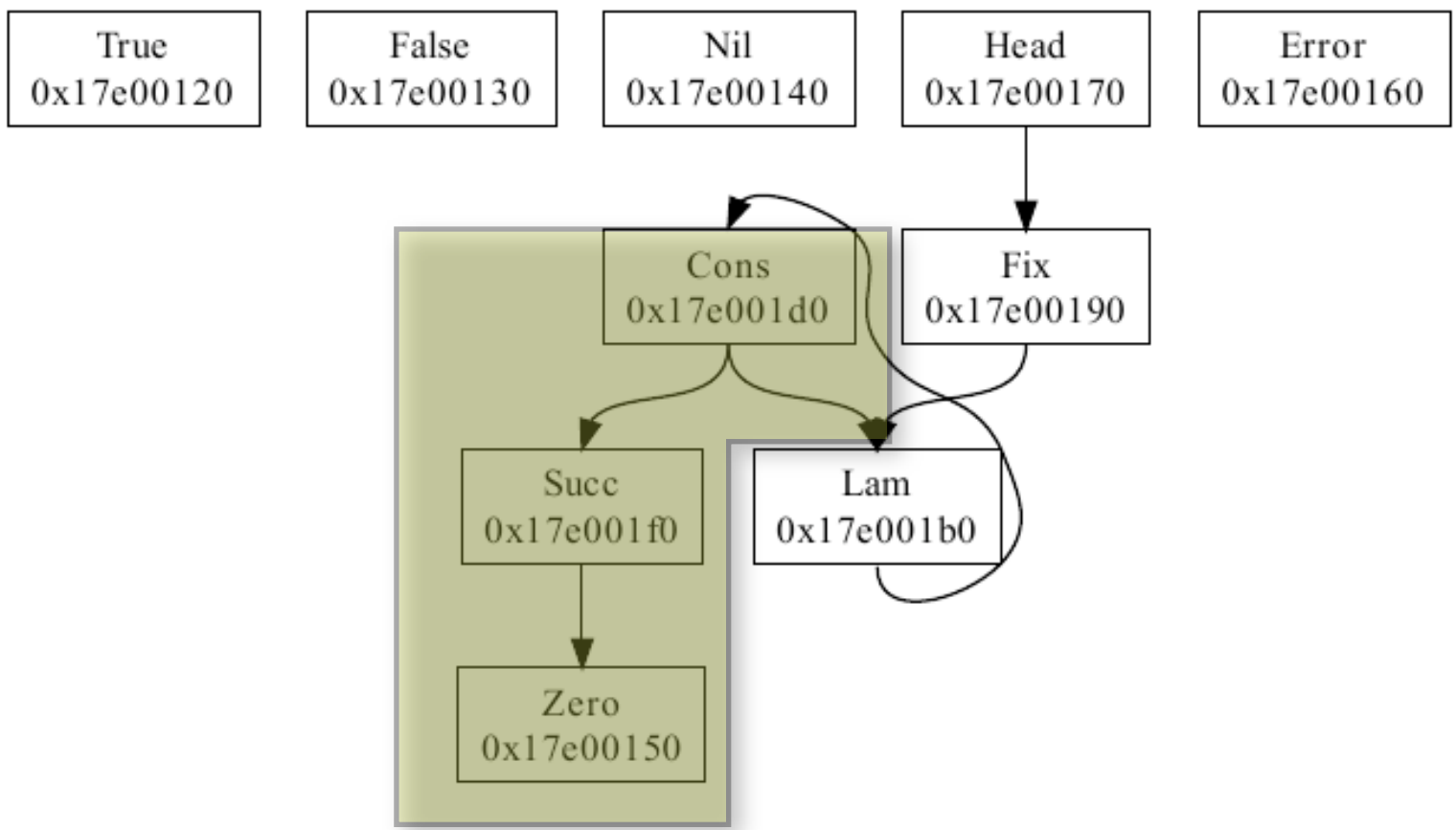

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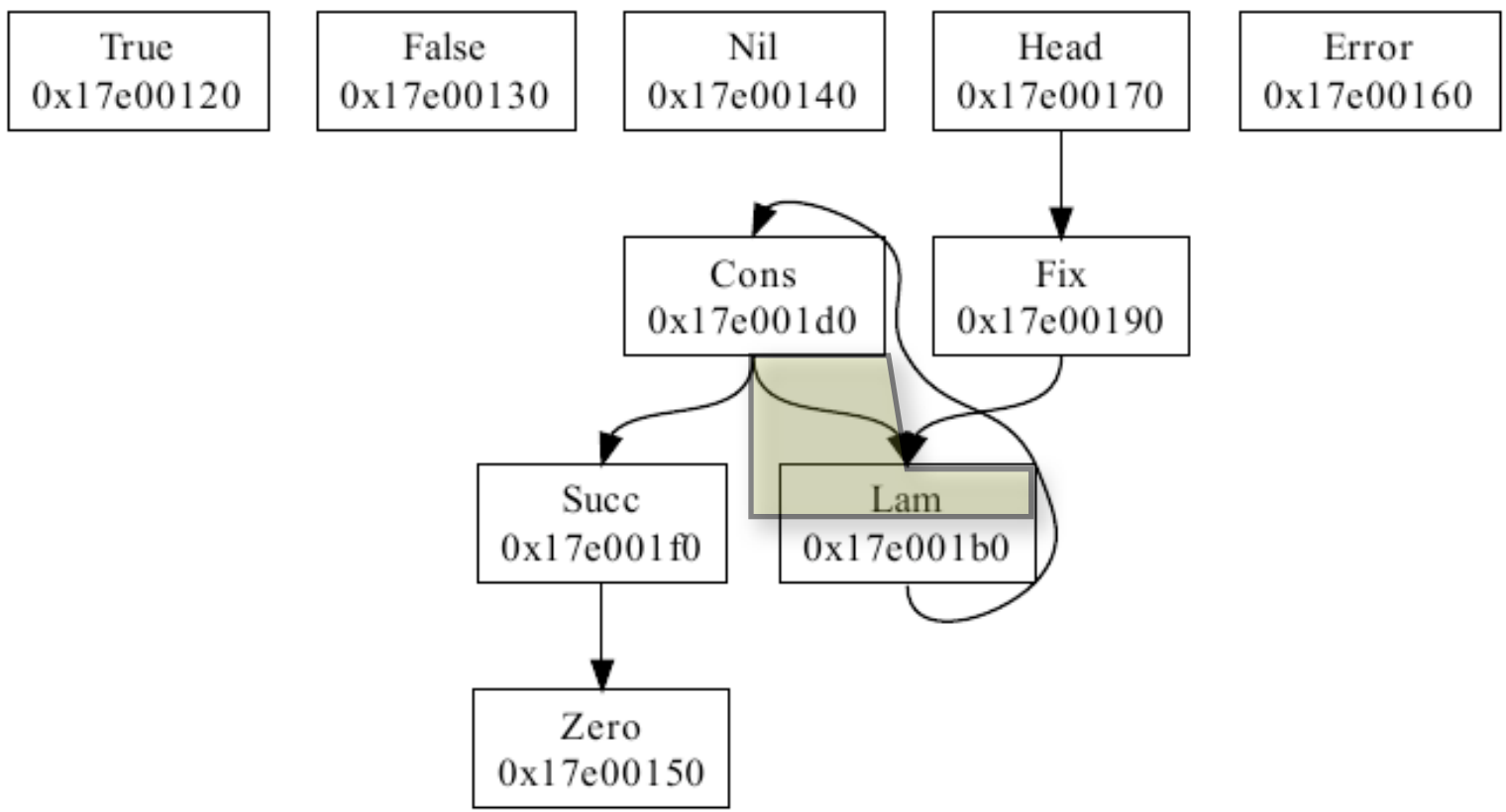
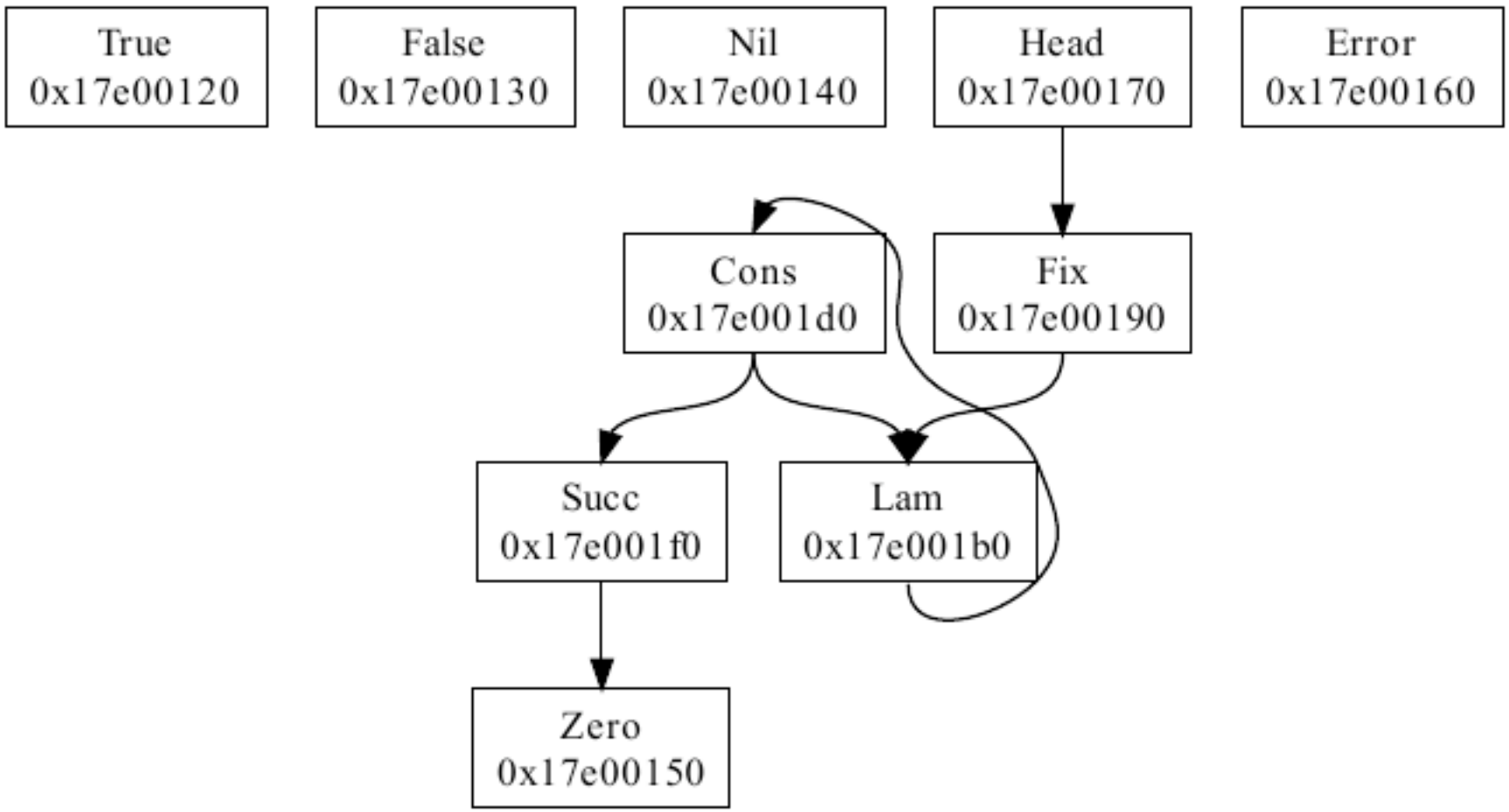


Tying the knot

- Step through the reduction of: `head (fix (\rec -> cons 1 rec end))`
- Observe the reduction of the fixed point carefully.
- The “infinite” list of ones becomes a cyclic graph.
- Normal order reduction ensures that the program terminates.
- Diagrams generated by GraphViz. http://graphviz.org/
- The compiled program writes “dot graph” files at each important step in reduction.

Step 1. The initial graph



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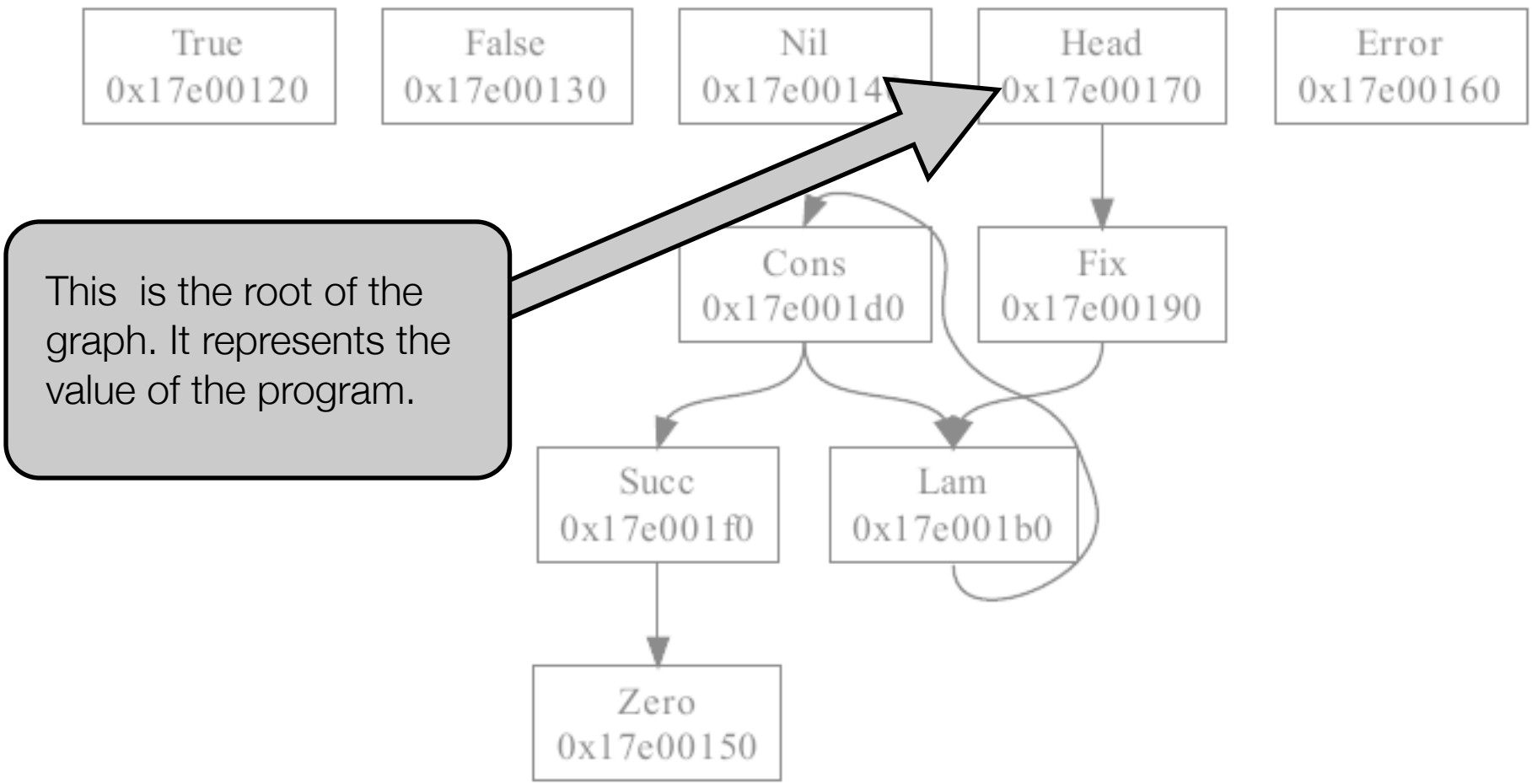
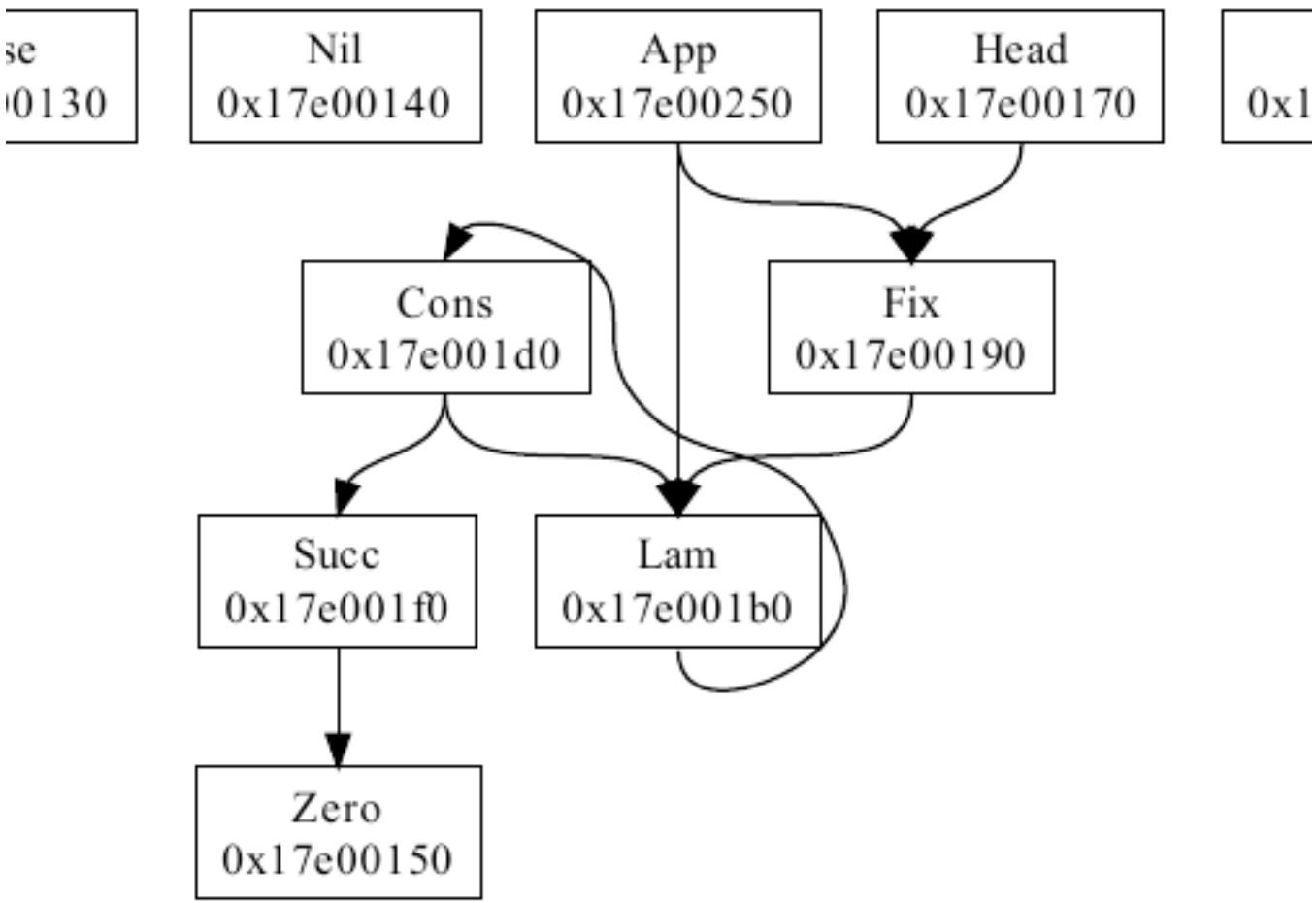


Step 2. unfold the fixed point



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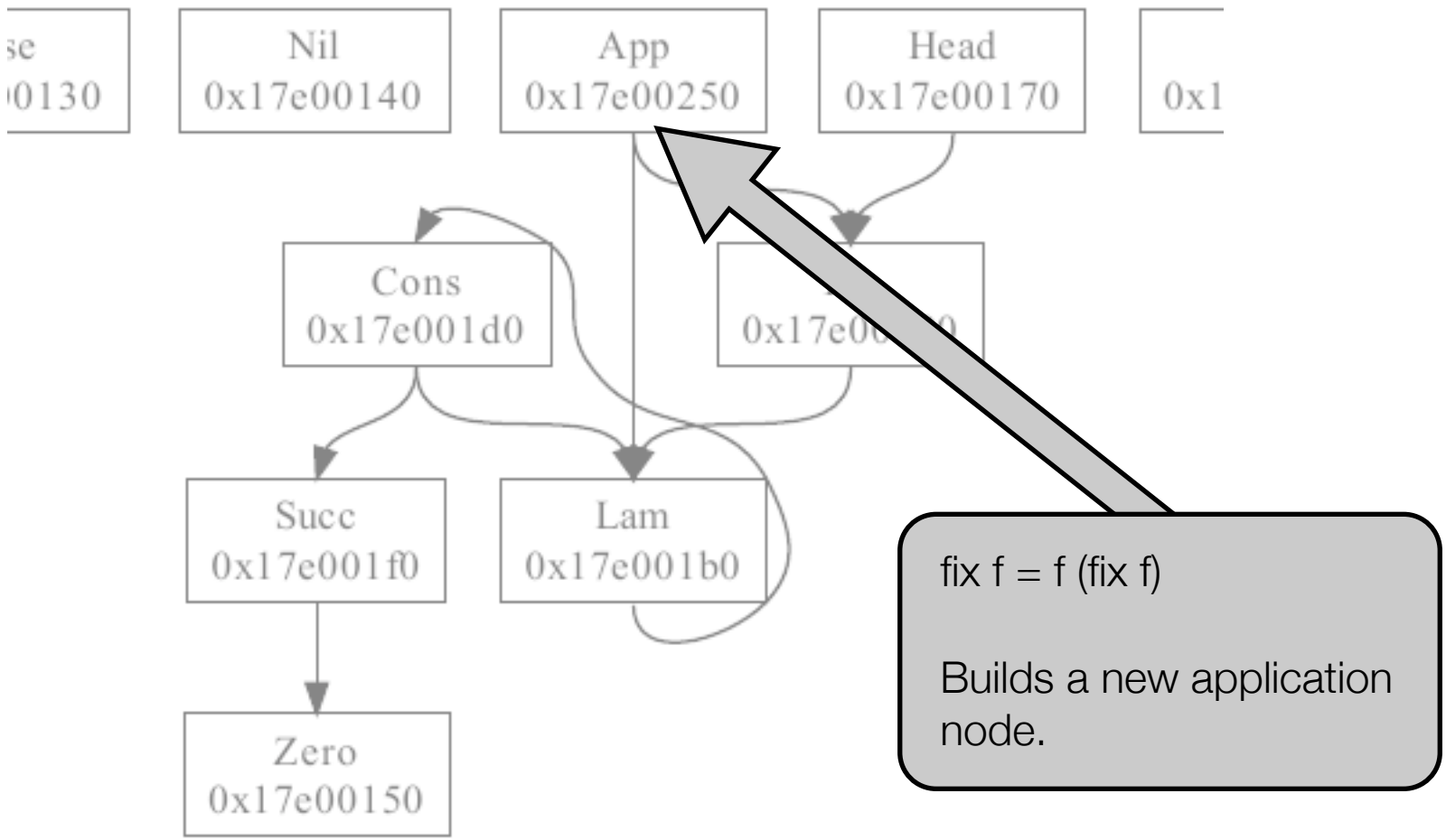
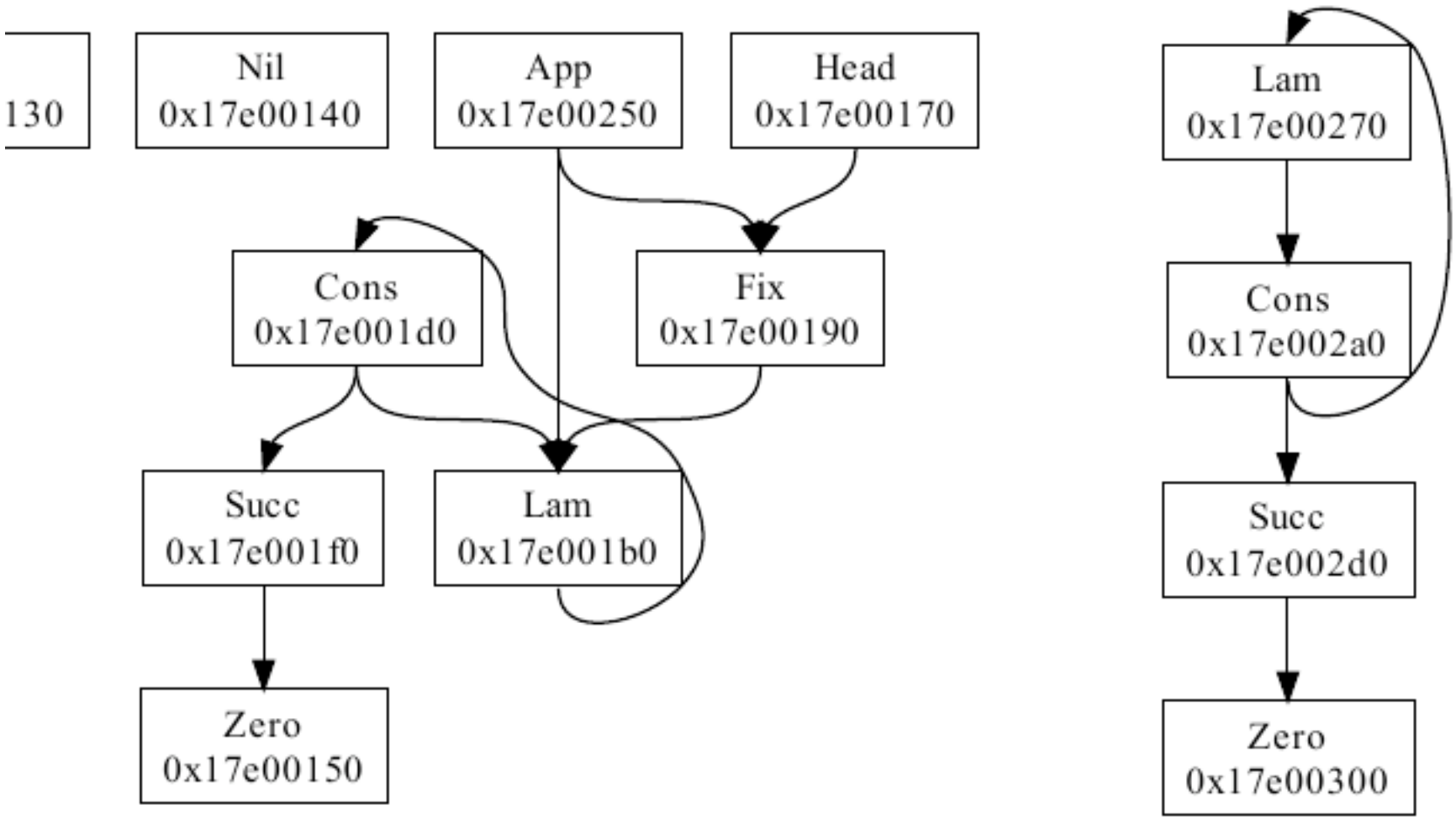


Step 3. copy the left child of the application



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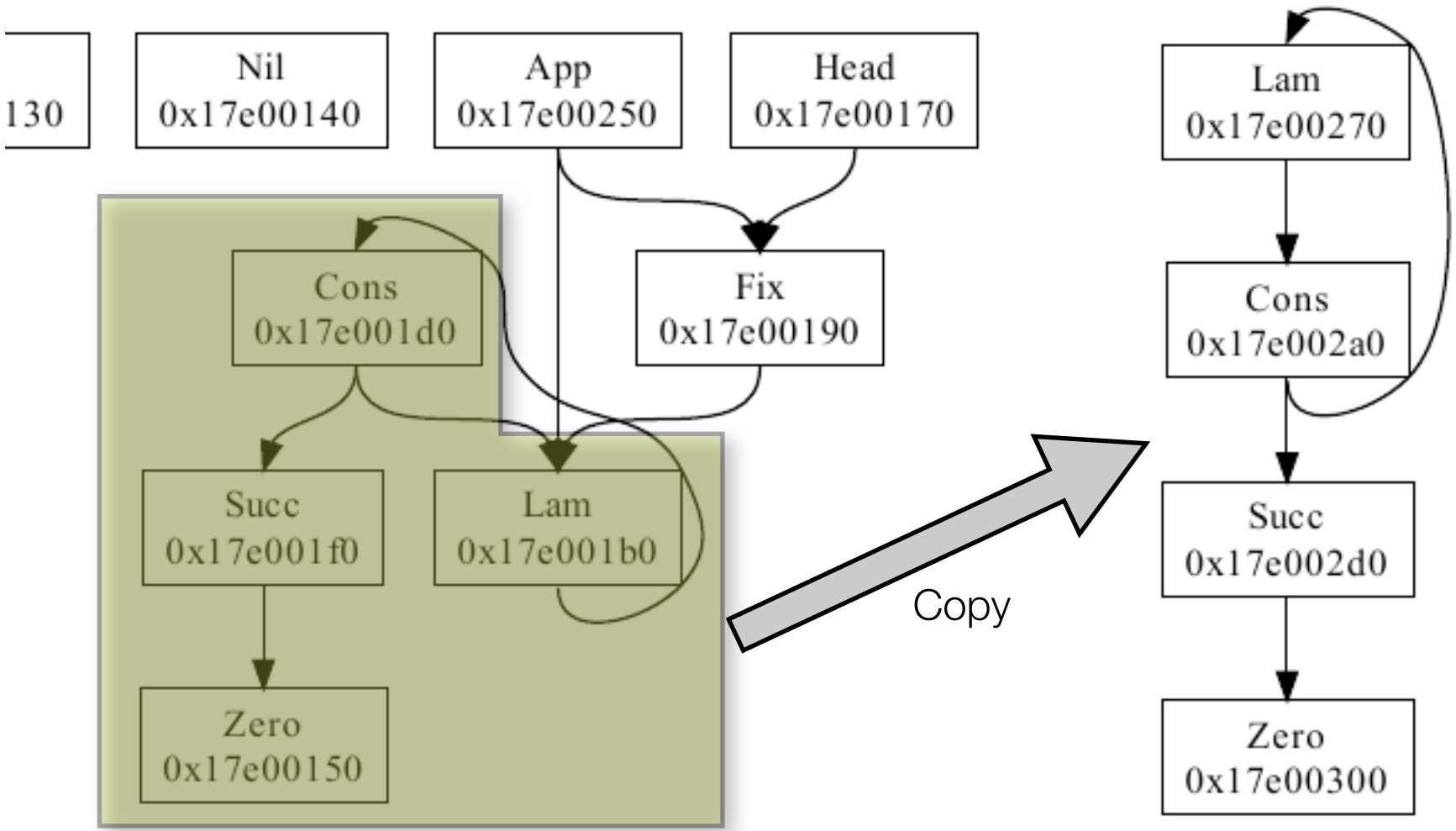
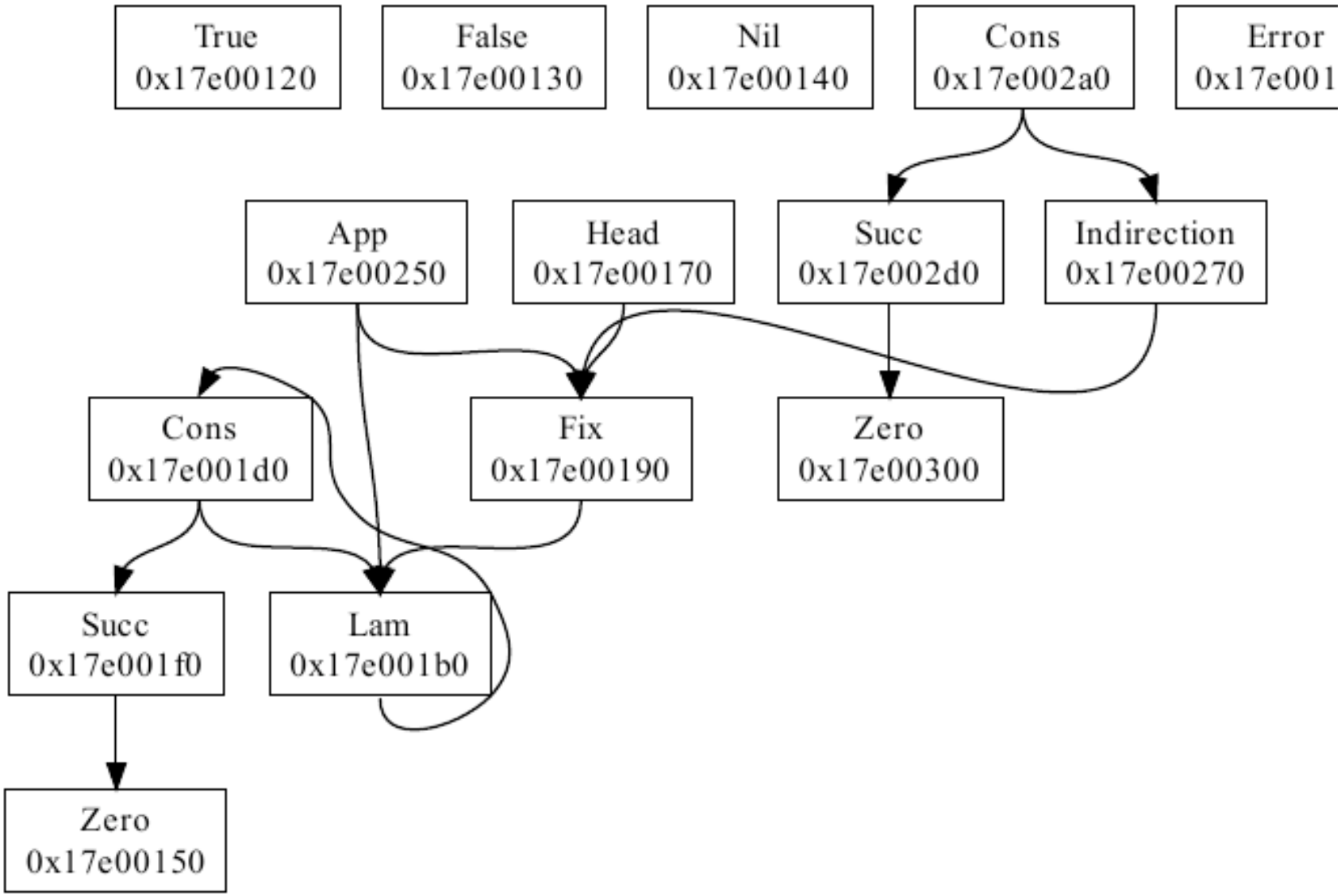


Step 4. substitute the parameter with the argument



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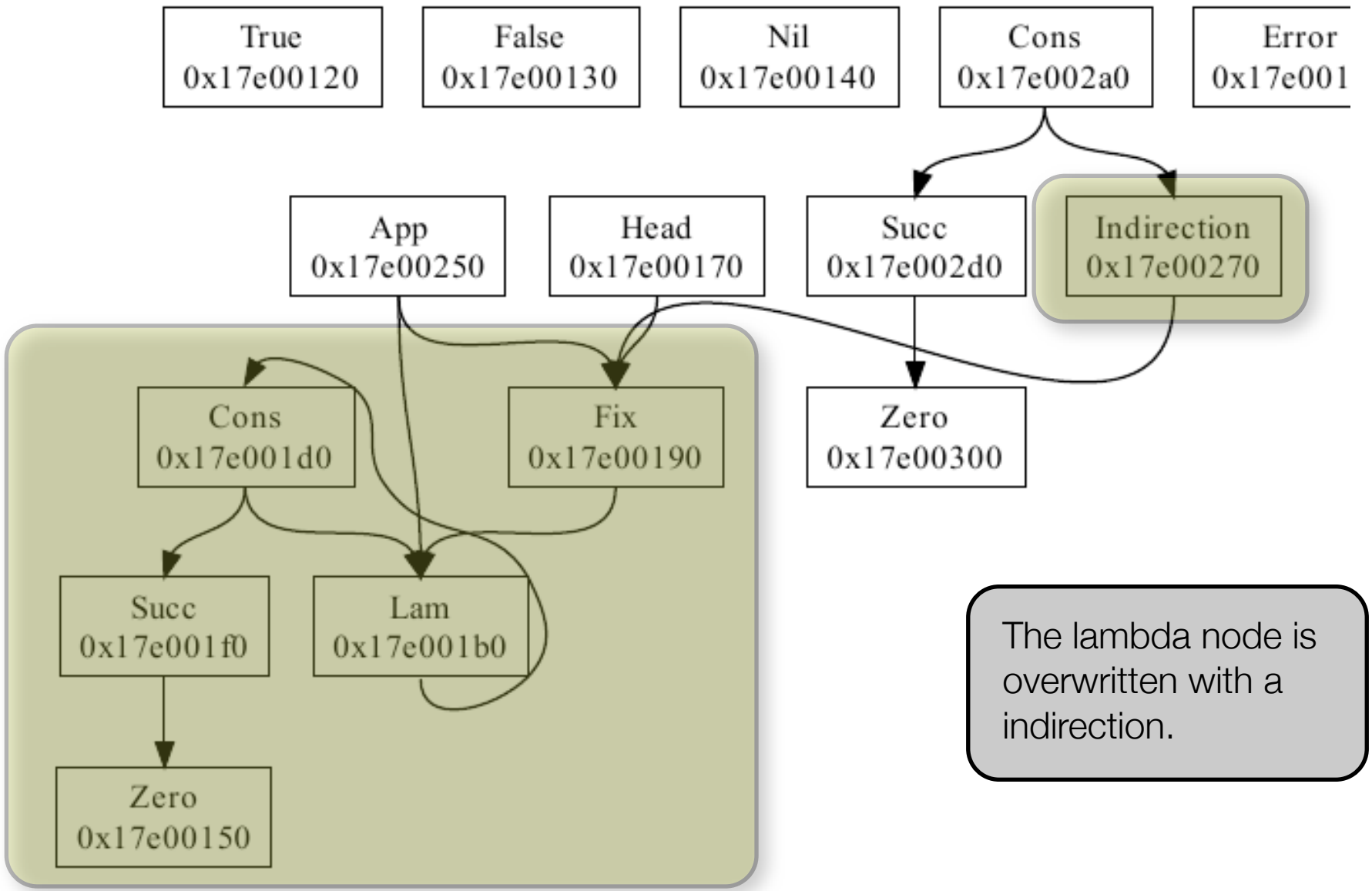
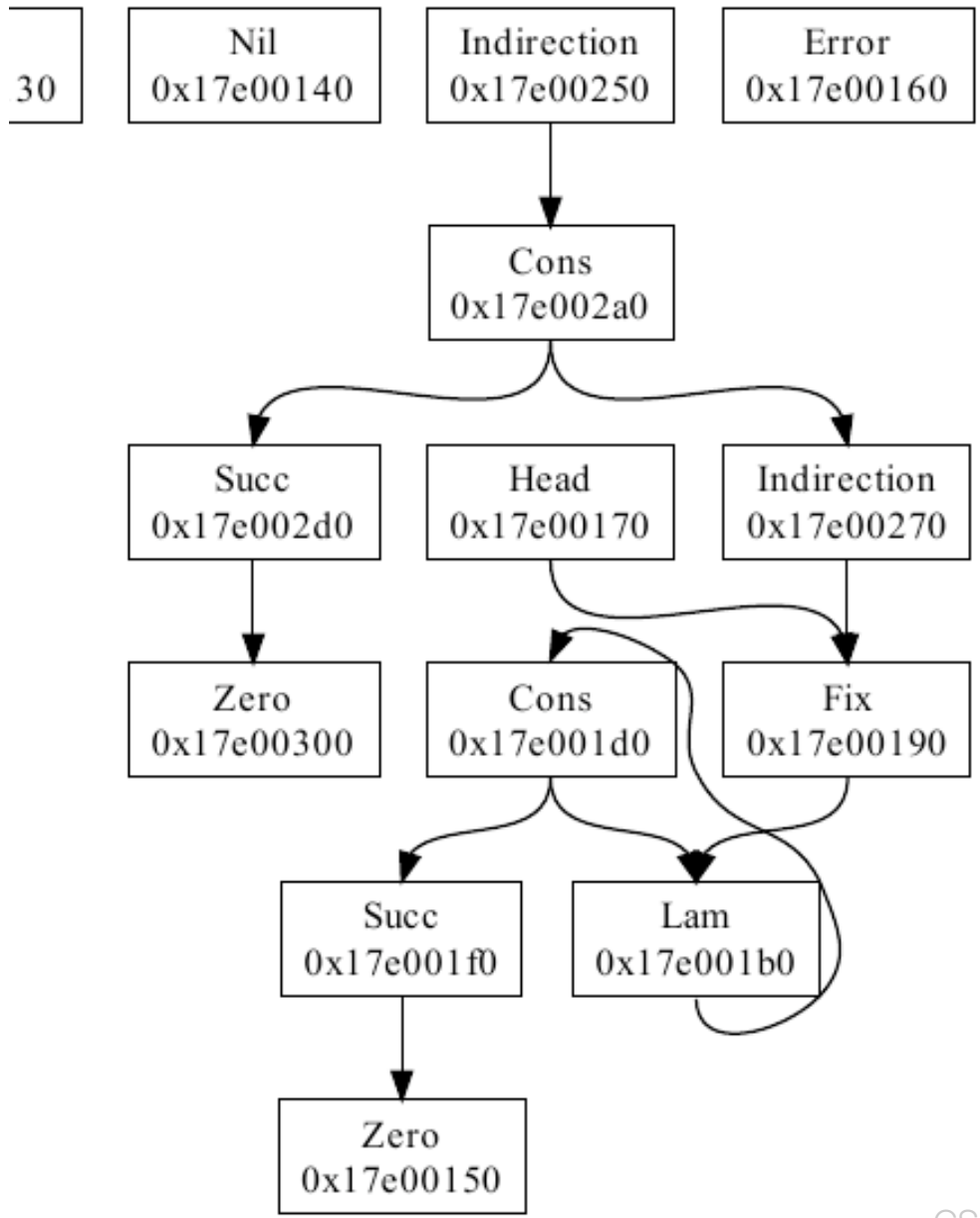


Step 5. overwrite the application node with its result



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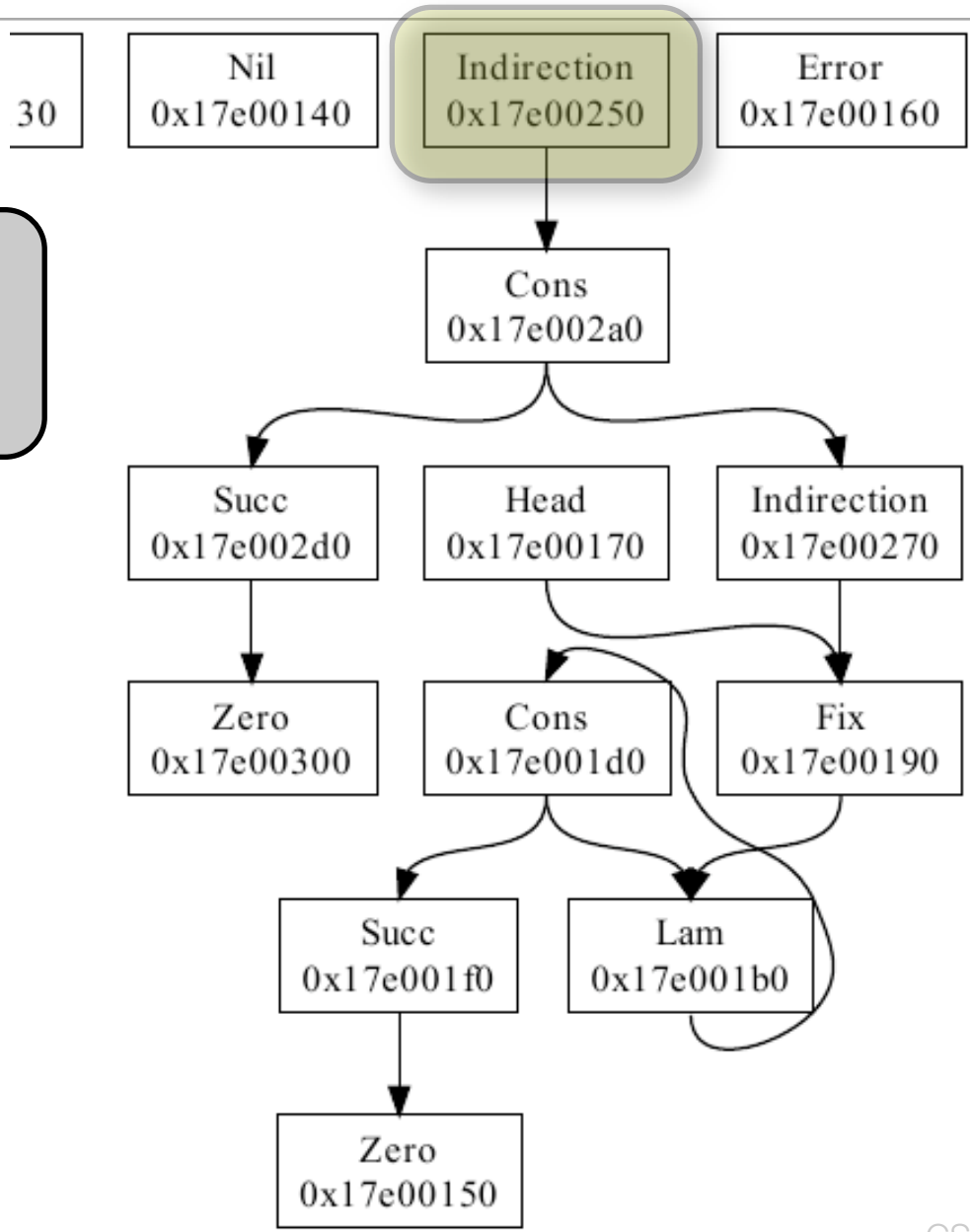
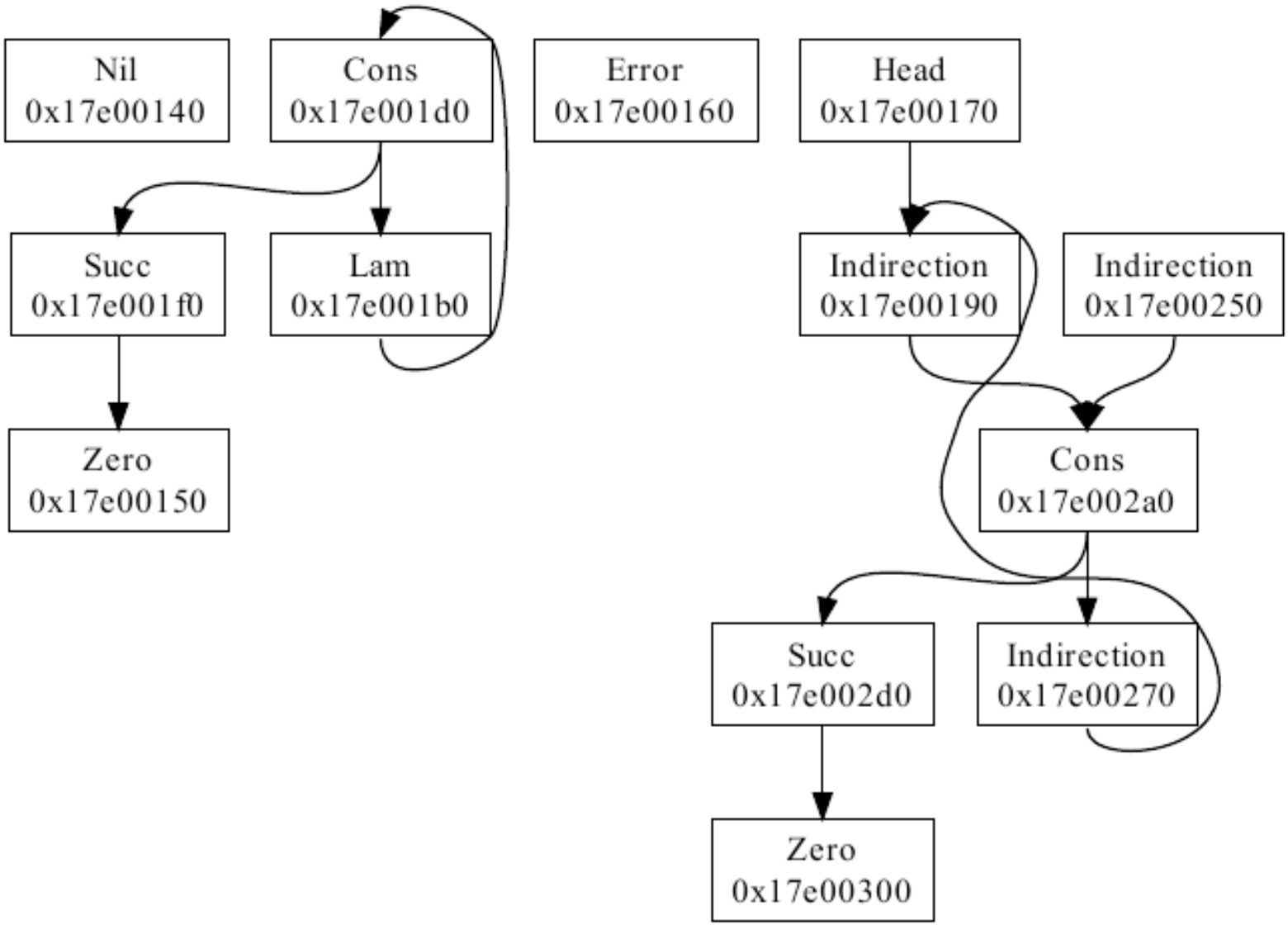
The application node is overwritten with a indirection.



Step 6. overwrite the fix node with its result



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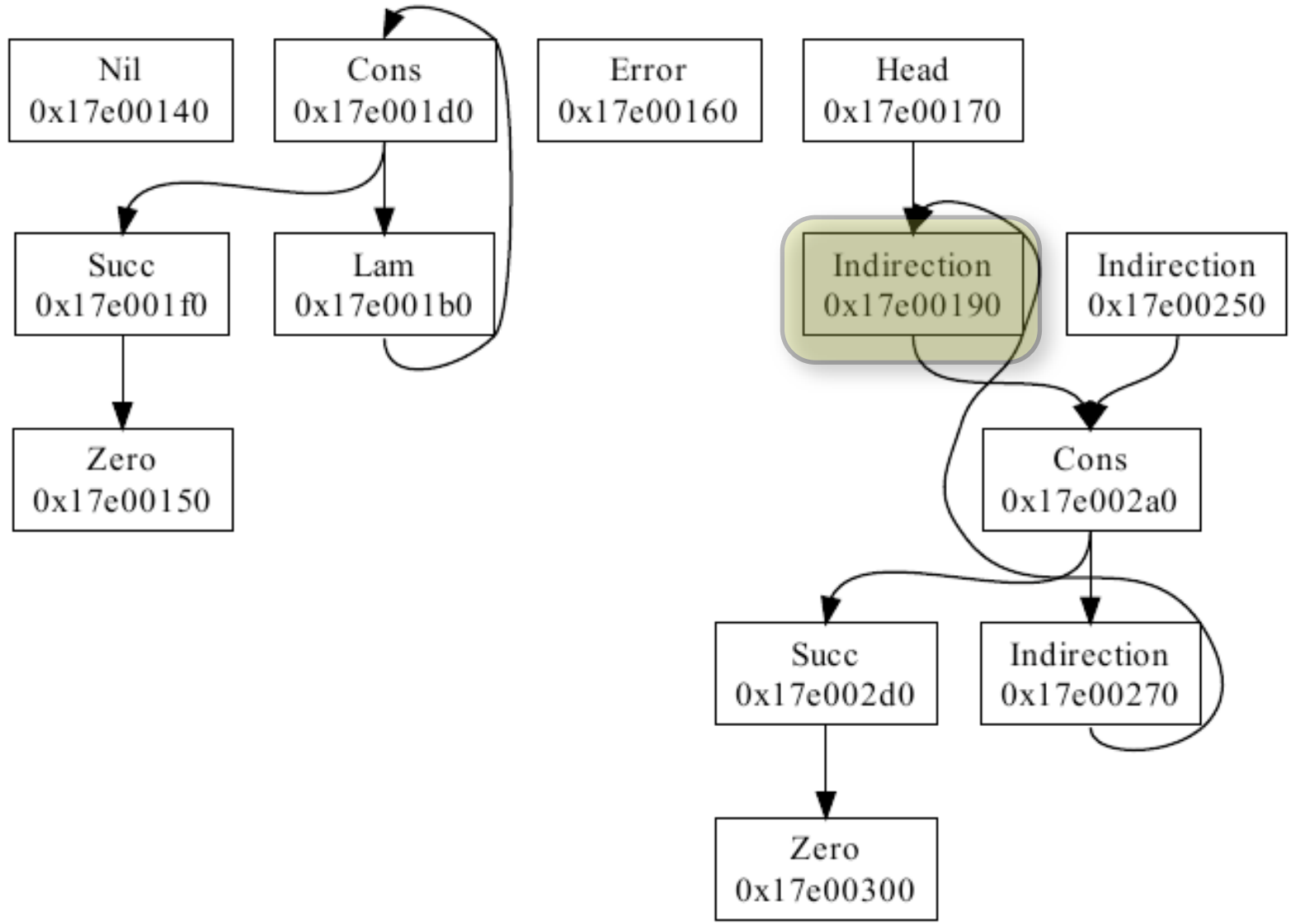

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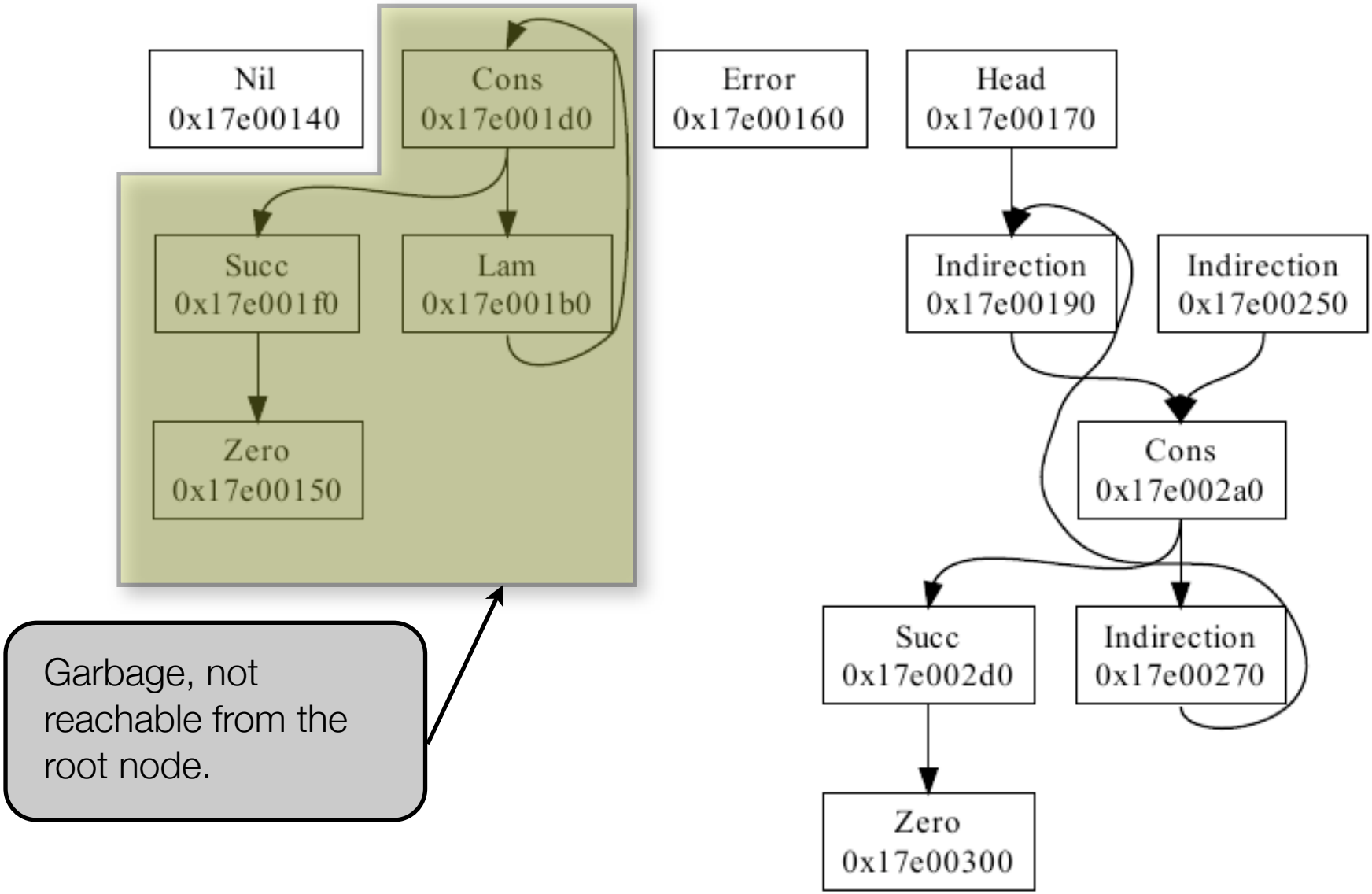

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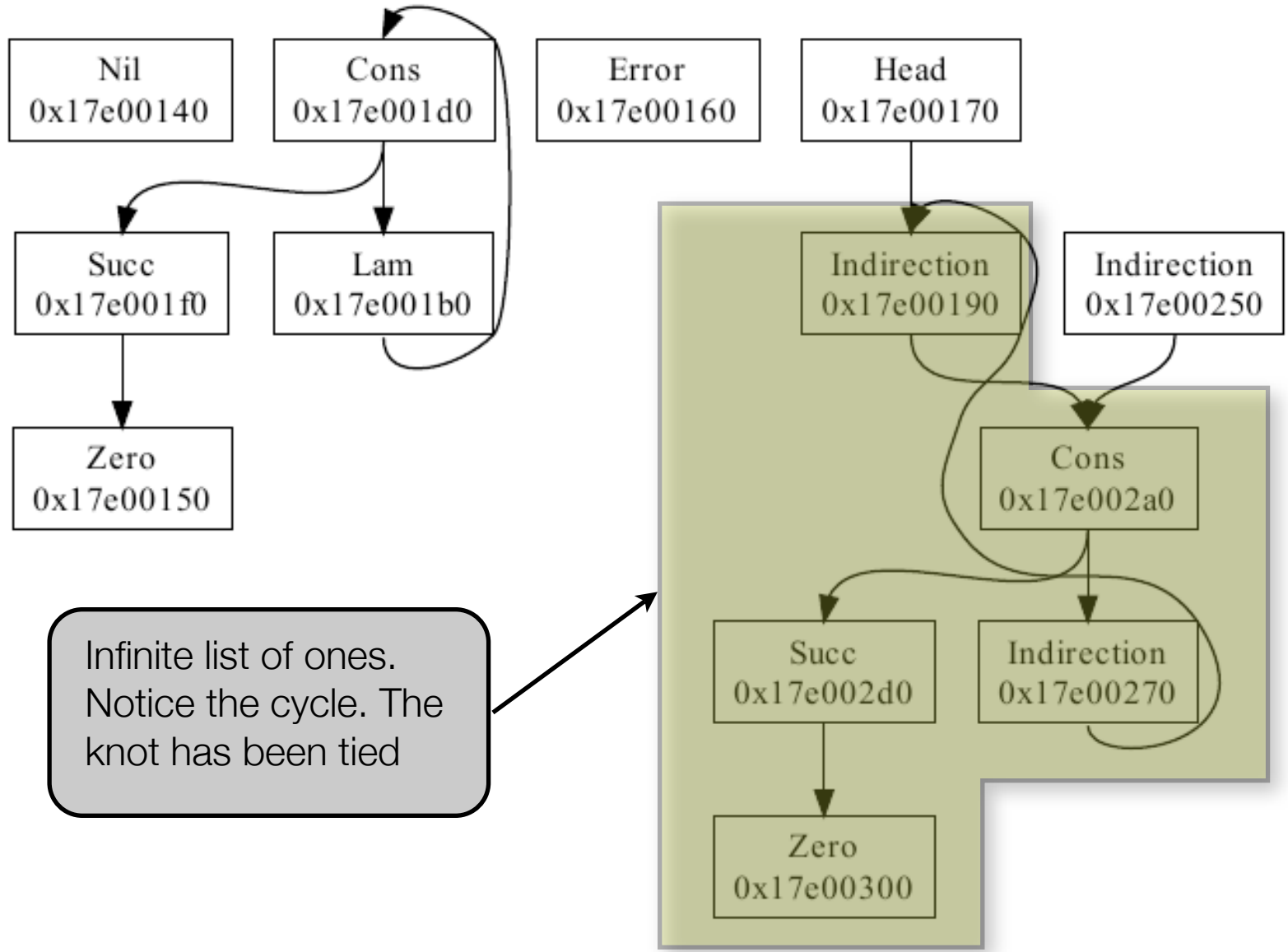

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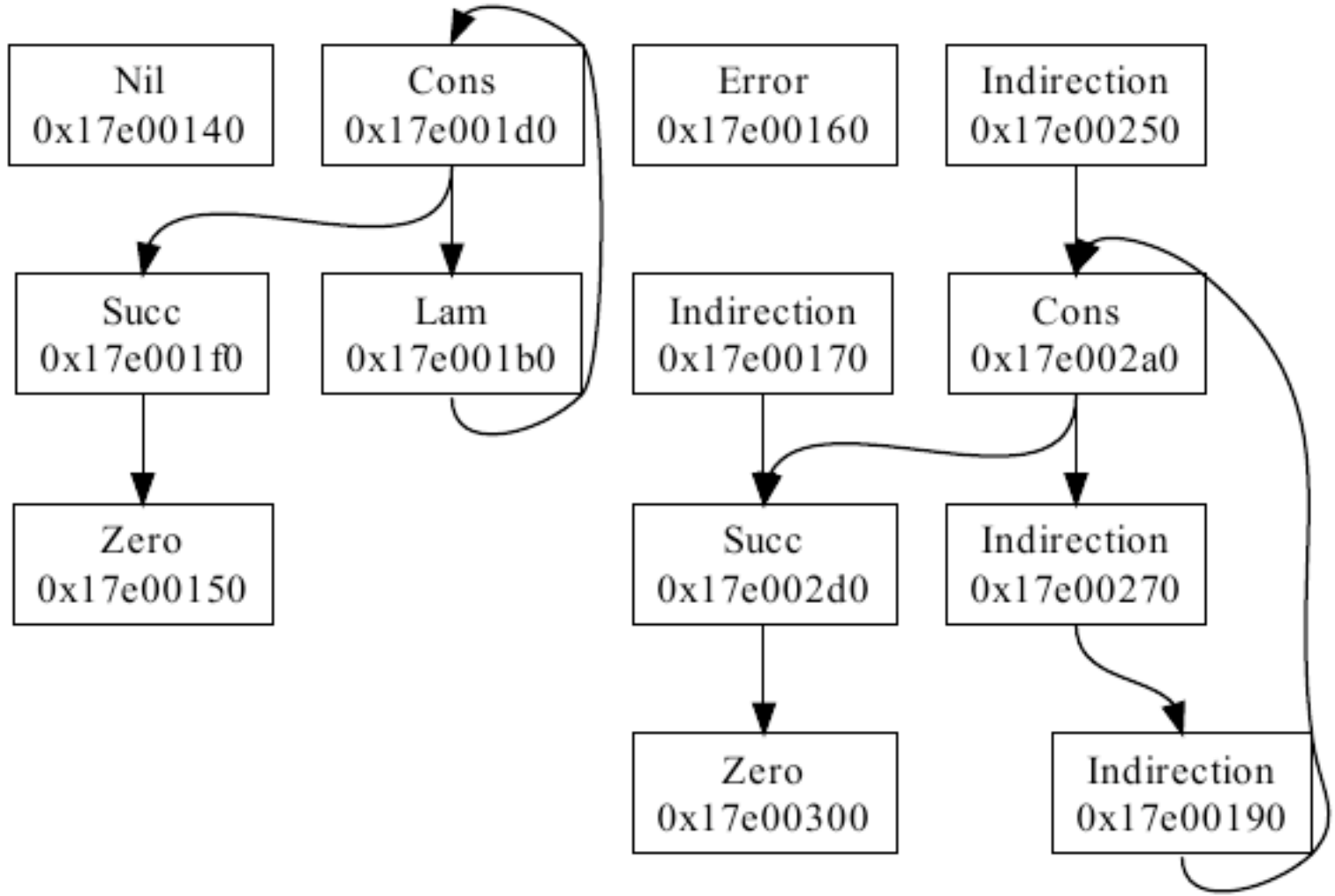

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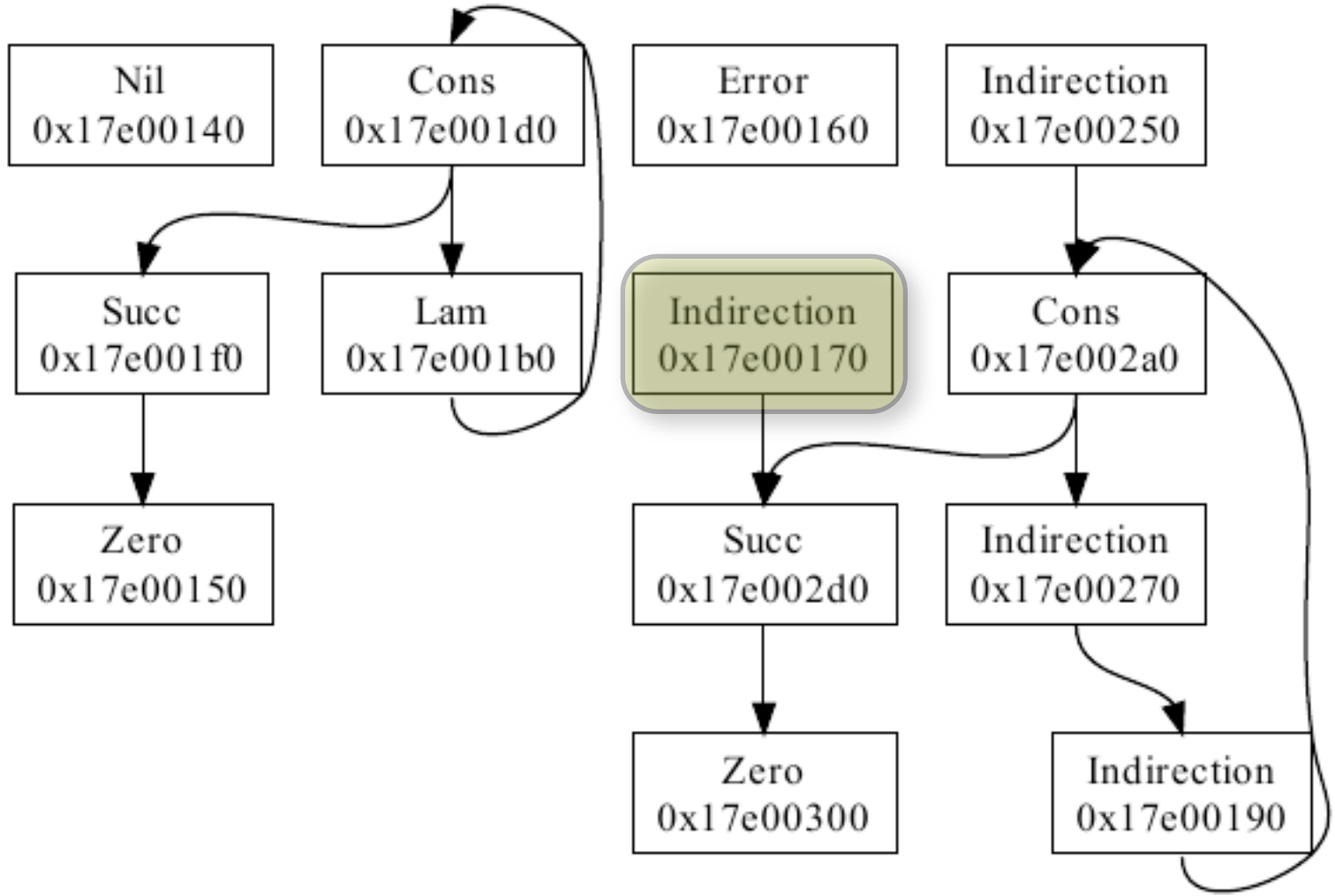

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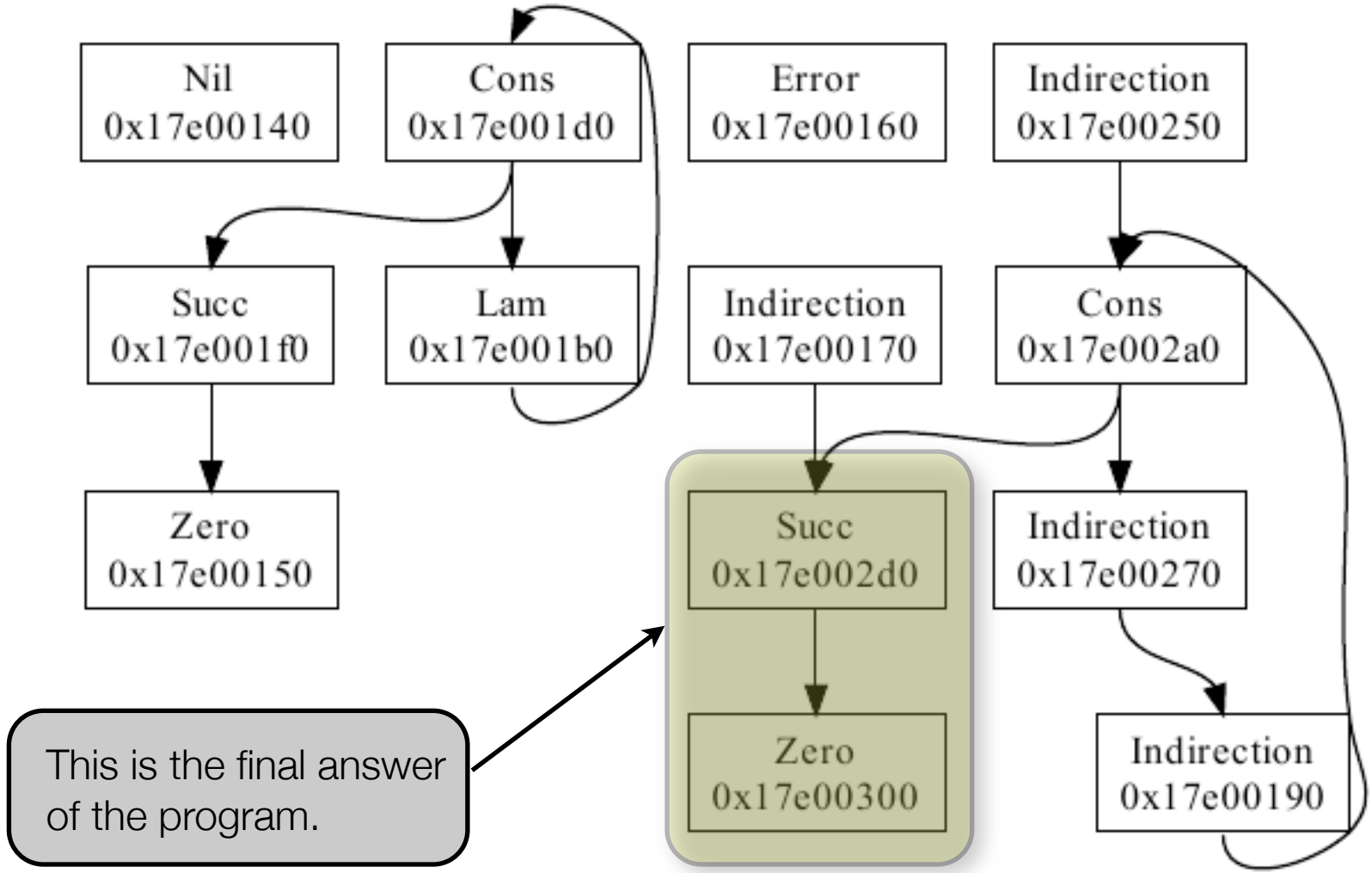

The compiler

- Translates miniFP programs to C code.
- The output C code is a function which builds the initial graph representation of the program as a data structure in the heap.
- The output C code is linked with a library which contains a reduction engine.
- The reduction engine rewrites the graph until it reaches WHNF (or it loops).
- The final value is pretty printed on standard output.

Compiled output

```
#include "build.h"
GraphPtr build (void)
{
   GraphPtr v_0 = HEAD;
   GraphPtr v_1 = FIX;
   GraphPtr v_2 = LAM;
   GraphPtr v_3 = CONS;
   GraphPtr v_4 = SUCC;
   EDGE(v_4,0,zero);
   EDGE(v_3,0,v_4);
   EDGE(v_3,1,v_2);
   EDGE(v_2,0,v_3);
   EDGE(v_1,0,v_2);
   EDGE(v_0,0,v_1);
   return v_0;
}
```

Reduction engine

```
GraphPtr reduce (GraphPtr g)
{
   GraphPtr result = NULL;

   if (is_whnf (g))
   {
      return g;
   }
   else
   {
      switch (g->tag)
      {
          /* ... reduce graph and build result ... */
      }
      indirect (g, result);
      return result;
   }
}
```

Selected Literature

- *Theorie des ensembles*, Bourbaki (1954) (according to Shivers and Wand).
- *Semantics and Pragmatics of the Lambda Calculus*, Wadsworth (1971).
- *The implementation of functional languages*, (Chapter 12) Peyton Jones (1987).
- *An algorithm for optimal lambda-calculus reduction*, Lamping (1990).
- *Lambda Calculi plus Letrec*, Ariola and Blom (1997).
- *Bottom up beta substitution*, Shivers and Wand (2004).
- *Lambda animator*, Mike Thyer (2007). http://thyer.name/lambda-animator/